Discrete geodesics

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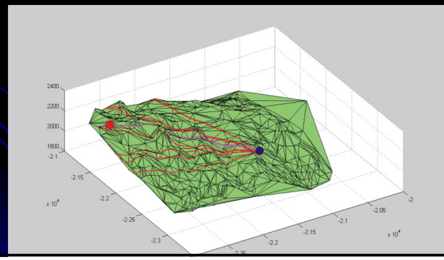
Motivation – The shortest path problem

- Problem: Given two points on the surface, find the shortest path between them.
- Geodesic – if there is the shortest line between two points, then it is the geodesic.



Contents

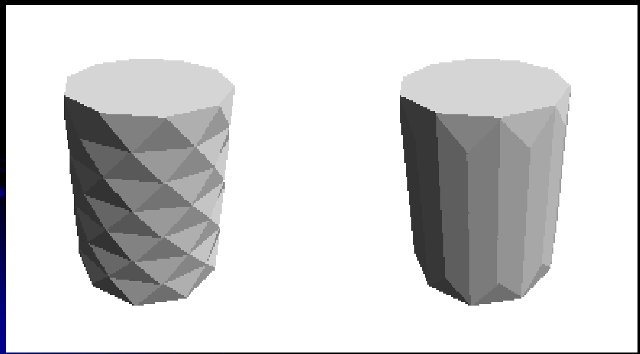
- Definition of discrete surfaces
- Discrete differential geometry
- Discrete geodesics
- How do the discrete surfaces affect the solution of the shortest path problem?

Discrete surfaces - definition

- ***Discrete surface*** = polyhedral model of a smooth surface, reconstructed from the given discrete data set, decomposed in triangles, quadrilaterals or other polygons.
- Model is ***adequate*** if the geometric characteristic of the source smooth model can be determined by computing its curvatures.

Discrete surfaces - types

Which triangulation describes the smooth source surface better?



Discrete differential geometry

How can be the following terms defined on the discrete surfaces?

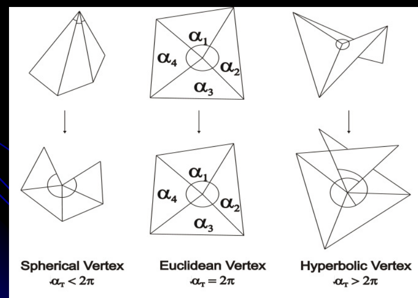
- Tangent plane of the surface and surface normal
- Direction on the surface
- Osculating plane of the curve
- The first and second curvature of the curve
- Normal curvature
- Asymptotic and main directions
- Gauss curvature and main curvature
- Geodesic curvature and geodesics
- Etc.

Discrete differential geometry – previous work

- There are many former results in discrete differential geometry, but…
- …the former results define the same term by different ways ("Each author defines his geodesic").
- Discrete geometry group TU Berlin

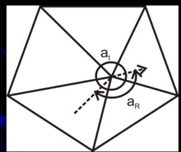
Gauss curvature

- On the smooth surface: $K = k_1 * k_2$
- On the discrete surface: $K = 360° - a_t$



Geodesic curvature

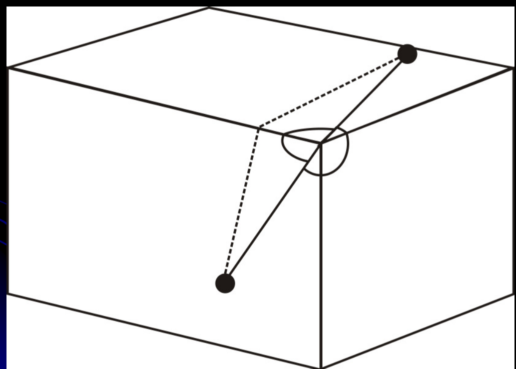
- On the smooth surface: k_g = |N (P' x P'')|
  - N is the normal vector of the surface
  - P' the unit tangent vector of the curve
  - P'' the vector of the first curvature of the curve
- On the discrete surface: $K_g = \frac{2\pi}{a_t}\left(\frac{a_t}{2} - a_R\right)$



Discrete geodesics

- Smooth surfaces: *J* is a geodesic if ***one*** of the following properties holds:
  1) *J* is locally the shortest curve
  2) *J* has vanishing geodesic curvature $K_g = 0$
  3) *J''* is parallel to the surface normal
- Discrete surfaces:
  - Ad 1) – the shortest geodesic
  - Ad 2) – the straightest geodesic
  - Ad 3) – not defined yet but could be – it will differ from the other ones in the hyperbolic vertices.

Discrete geodesics

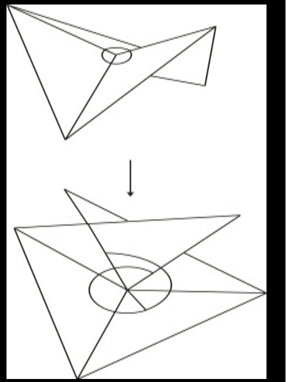


How do the discrete geodesics differ?

- In the triangles, edges and Euclidean vertices – they are the same
- In the spherical vertices – the shortest geodesic never goes through the spherical vertex!
- In the hyperbolical vertices

Hyperbolic vertex and discrete geodesic

- There are infinitely many shortest geodesics with the same input direction going through the hyperbolic vertex.
- There is a shadow in the neighbourhood of the hyperbolic vertex where two points can not be connected with the straightest geodesic.



How do the discrete surfaces affect the solution of the shortest path problem?

- The shortest path cannot go through the spherical vertices (except of its first and end points)
- The straightest geodesic is impractical for the shortest path problem because of its characteristics in the neighbourhood of the hyperbolic vertices.
- The success depends also on the precise choice of the appropriate triangulation (on the type of the discrete surface)

Future work

- What is the best (data-dependent) triangulation for the shortest path problem?
- Can the third type of geodesic solve the shortest path problem?
- Can we estimate the area where will the shortest path lead if we know the curvatures of the surface?

References

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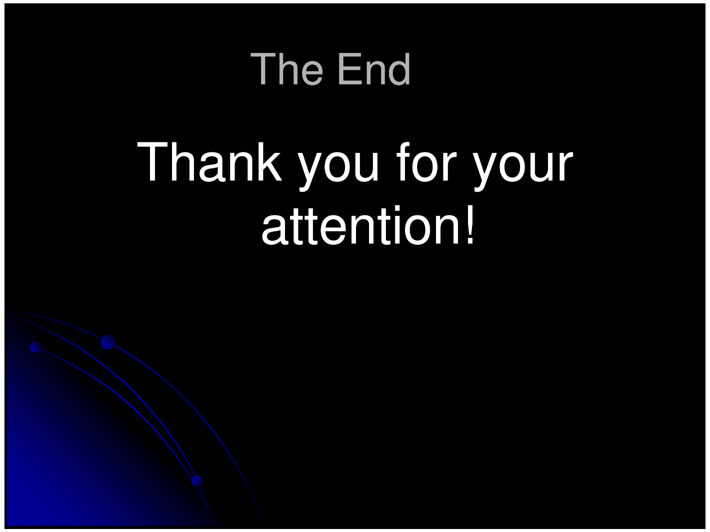
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Thank you for your attention!