Shortest path

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Content

- Types and algorithms for the shortest path
- Introducing an older algorithm for the shortest path computing using geodesic
- My proposal of its improvement

Shortest path

- Euclidean shortest path
  - Find the shortest path from *s* to *t* between the obstacles.
  - NP-hard
- Geodesic shortest path
  - Find the shortest path ON THE SURFACE from *s* to *t*.
  - $O(n^2)$

Algorithms

- Classification by computation
  - Exact
  - Approximative

Approximation algorithms: The cost of the path is $(1+\varepsilon)$-times the cost of an exact shortest path.

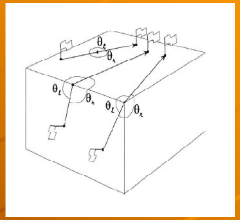
Polyhedral Surface	Cost Metric	Approx. Ratio	Time Complexity	Reference
Convex	Euclidean	1	$O(n^3 \log n)$	Sharir and Schorr (1986)
Non-convex	Euclidean	1	$O(n^2 \log n)$	Mitchell et al. (1987)
Non-convex	Euclidean	1	$O(n^2)$	Chen and Han (1996)
Non-convex	Euclidean	1	$O(n \log^2 n)$	Kapoor (1999)
Convex	Euclidean	2	$O(n)$	Hershberger and Suri (1995)
Convex	Euclidean	$1+\varepsilon$	$O(n \log(1/\varepsilon)+1/\varepsilon^3)$	Agarwal et al. (1997)
Convex	Euclidean	$1+\varepsilon$	$O(n+\frac{\log n}{\varepsilon^{1,5}}+\frac{1}{\varepsilon^3})$	Har-Peled (1999)
Convex	Euclidean	$1+\varepsilon$	$O(\frac{n}{\sqrt{\varepsilon}}+\frac{1}{\varepsilon^4})$	Agarwal et al. (2002)
Convex	Euclidean	$1+\varepsilon$	$O(\frac{\sqrt{n}}{\varepsilon^{1,25}}+f(\varepsilon^{-1,25}))$	Chazelle et al. (2003)
Non-convex	Euclidean	$1+\varepsilon$	$O(n^2 \log n+\frac{n}{\varepsilon}\log\frac{1}{\varepsilon}\log\frac{n}{\varepsilon})$	Har-Peled (1999)
Non-convex	Euclidean	$7(1+\varepsilon)$	$O(n^{\frac{5}{3}} \log^{\frac{5}{3}} n)$	Varadarajan and Agarwal (2000)
Non-convex	Euclidean	$15(1+\varepsilon)$	$O(n^{\frac{8}{5}} \log^{\frac{8}{5}} n)$	Varadarajan and Agarwal (2000)

Computing the shortest path using geodesics

- Discrete geodesic
- An older algorithm
- Acceleration of the algorithm
- Results

Discrete geodesic

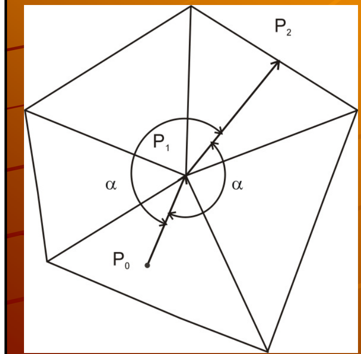
- The straightest curve on the surface (need not to be the shortest one)
- The left and right angles are identical

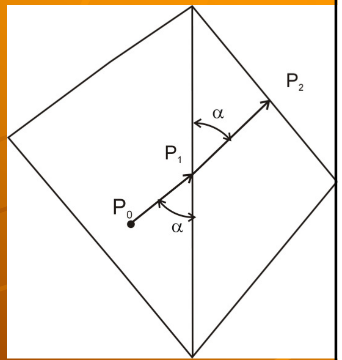


Computing a geodesic

- Algorithm was described in CAGD 2003
- Implemented in my diploma thesis
- Works on the triangulated surface.
- Given a start point and an initial direction, find the geodesic on the surface.

Computing a geodesic





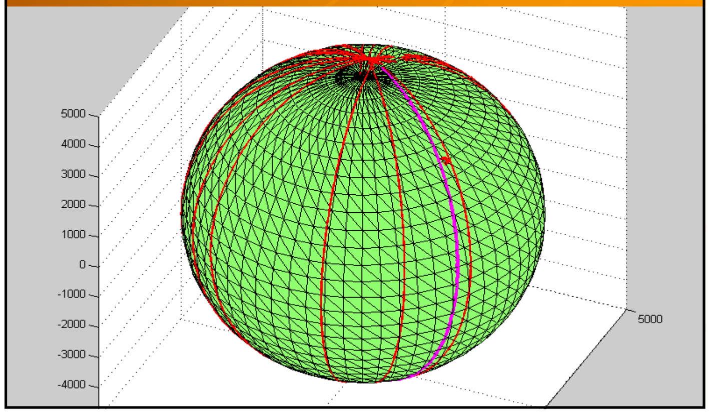
Computing the shortest path

- Given two points *s* and *t*, find the discrete geodesic λ which satisfies:
  - $\lambda(0) = s$
  - $\lambda\,(\text{length}\,(\lambda_{st})) = t$
  - $\lambda'(0) = \mathbf{v}$
  - $\text{length}\,(\lambda_{st}) = \min$

Computing the shortest path

- Algorithm implemented by Ing. Zábranský (2005):
  - The former introduced algorithm for computing the geodesic was used.
  - The initial vector is chosen randomly, then the algorithm runs iterative
- Result:
  - After c. 500 iterations, it chooses the curve that approximates best the shortest path between *s* and *t*
  - the relative error is about 0.01 after 200 iterations

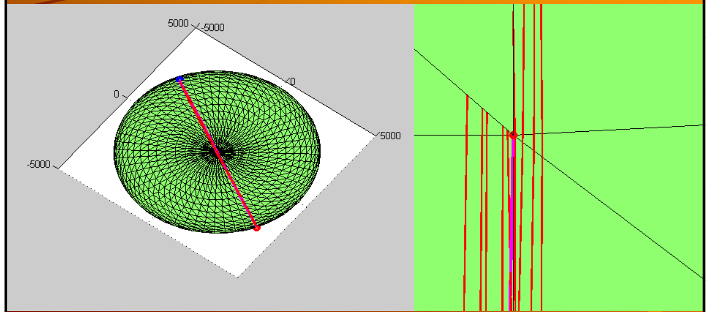
Demonstration for 20 iterations



My proposal to acceleration

- 1) The initial direction: v = t – s
- 2) Try changing the direction by a small angle.
- Results:
  - Relative error after 50 iterations is less than 0.0001
  - The best approximation of the shortest path is found much faster

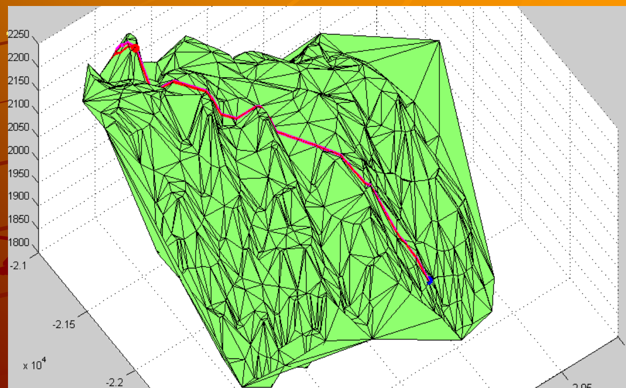
Accelerated algorithm for 10 iterations



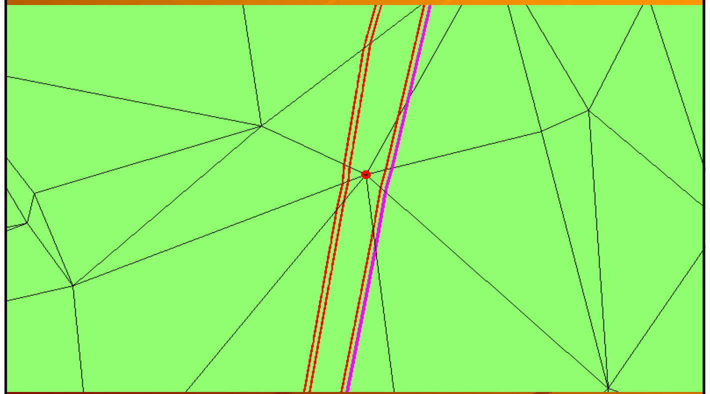
Comparison to the other algorithms

- Complexity: *O(k n)*
  - *k* number of directions
  - *n* number of triangles
- Advantages:
  - Fast
  - Simple
- Disadvantages:
  - Does not pass through the second point *t* exactly.

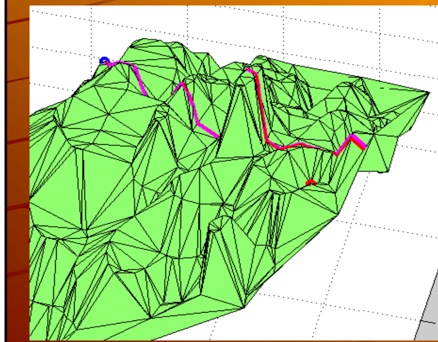
Demonstration of a complicated surface (for 10 directions)



Demonstration of a complicated surface



Demonstration of a complicated surface II.



- The relative error = 0.1137

Future work

- To improve the algorithm for non-convex surfaces
- To use methods of unfolding the surface